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Desmos Graphical Analysis of Binary Stars

A Tutorial with HU 481 (WDS 16212+2259) Example

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Abstract This tutorial describes the basic concepts behind binary stars, Kepler's Three Laws of Planetary Motion (and binary star motion), and the vital role binary star orbits play in determining stellar mass. The primary source of past published binary star observations, the *Washington Double Star Catalog,* is illustrated with an example, HU 481 (WDS 1612+2259). An image of the apparent orbit of HU 481, published in the *Sixth Catalog of Orbits of Visual Binary Stars,* is described in detail, along with instructions on inserting it as a background into Desmos that, via rotation, scaling, and translation, precisely matches Desmos' Cartesian coordinates. Observations of HU 481 not included in these graphs were added via Desmos. An apparent orbit ellipse was created in Desmos that matched the published apparent ellipse, while a new ellipse was created that better matched recent observations of HU 480, as they appeared to be going off the published orbit's track.

1. Binary Stars

Binary stars are gravitationally bound pairs of stars that rotate around their common center of mass. *Astrometry* "… a branch of *astronomy* that involves precise measurements of the positions and movements of stars and other celestial bodies" (Wikipedia 2023a) is used to analyze the motions of the two stars over time. Johannes Kepler's Three Laws of Planetary Motion, which he developed to describe our solar system, also apply to binary stars. Kepler's First Law establishes that the shape of the orbit is an ellipse. His Second Law states that the speed of each star varies throughout its orbit, with equal areas being swept out in equal times. Kepler's third law—which relates the period of rotation, the semi-major axis of the ellipse, and the mass of the two stars—can then be used to determine the mass of the two stars, once the period and semi-major axis are established via observations and analysis.

$$(m_1 + m_2) = a^3 / P^2$$

where
m_1 and m_2 are the masses of the primary and secondary stars, respectively, in solar masses
a is the semi-major axis of the true ellipse in astronomical units
P is the period in years

Binaries have been observed at high magnification with telescopes since 1781 (Herschel 1785). Each observation consists of measuring the angle the secondary (fainter) star makes with respect to the primary (brighter) star and celestial north (the position angle in degrees), and the separation between the two stars (in arc seconds, an angular measurement) as shown in Figure 1. The observed position angles and separations~~,~~ gathered over decades or centuries, can then be used to determine the parameters of an apparent elliptical orbit via Kepler's First Law.

Kepler's Second Law can then be used to transform an apparent orbit into a true orbit, while Kepler's Third Law can be used to calculate the sum of the stellar masses ($m_1 + m_2$). Kepler's three laws are summarized in Figure 2. Parsing this sum into individual masses is the last step in this long process, and is usually accomplished through the analysis of a radial velocity curve of the binary (a topic beyond the scope of this paper). The end result—the accurate knowledge of stellar masses made possible by the orbital analysis of binaries—is *the* key to understanding the life cycles of stars, i.e., how they evolve over time.

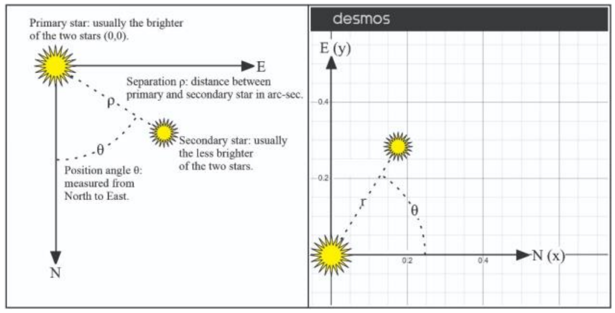

Figure 1: Position angle and separation for a binary star.

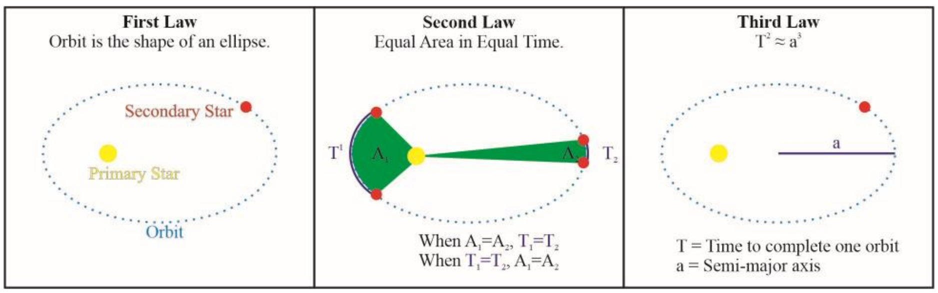

Figure 2: Illustrations of Kepler's Three Laws of Planetary Motion.

For many decades, the United States Naval Observatory (USNO) has served the international astronomical community by compiling all published observations of double stars into the Washington Double Star (WDS) Catalog (2023). Stars that at first appear to be single can, when viewed through a telescope at high magnification, be seen as two stars. Many of these double stars are far apart in space and only appear close together in the sky because of their chance alignment along the line of sight from Earth. These are referred to as *optical doubles* and are of limited scientific interest. However, as discovered by William Herschel (1803), some of these stars are not only close together, but they are rotating around a common center of gravity. Herschel named these gravitationally bound stellar pairs *binaries.*

Binaries fall into two major categories: *suspected* and *confirmed* binaries. Suspected binaries *may* be gravitationally bound, but no analytically derived equation for their elliptical orbit has yet been published for any of several reasons: there may be too few observations to compute an orbit, not much of an orbit has been observed yet, or the points are too scattered or confusing to make the orbital computation and publication worthwhile.

Confirmed (known) binaries, on the other hand, have published orbits. As another service to the international astronomical community, the USNO publishes the Sixth Catalog of Orbits of Visual Binary Stars (Matson et al. 2023). This catalog, often referred to as the 6th Orbit Catalog, provides a binary's *orbital elements.* These seven elements are the parameter values for the ellipse equation that shows the true orbit as it would be seen if it were viewed face-on from outer space, as well as parameters that describe how the orbit is tilted with respect to the line-of-sight from Earth. This catalog also provides a plot (graph) of the tilted ellipse (the apparent orbit) as seen from Earth. This plot allows one to tell, at a glance, how well the apparent ellipse fits the observations.

The Desmos graphing calculator routine described in this paper analyzes the apparent orbits, assumed to be elliptical per Kepler's First Law. The challenge in this analysis is creating an ellipse that matches the observations, i.e., matches the plotted points which are the changing position angles and separations between the two stars in the binary pair over time. This Desmos routine provides five sliders—one for each of the five parameters that describe a translated and rotated ellipse in the Desmos Cartesian plane. These sliders can be iteratively adjusted until an apparent ellipse is found that is a good match to the observations.

To use Kepler's Third Law to determine stellar mass, we must know the orbital period and the semi-major axis of the true orbit. Fortunately, Kepler's Second Law—equal areas being swept out in equal times—can be used to transform an apparent orbit into a true orbit as it would be seen if it were viewed face-on from outer space. This transformation requires solving Kepler's Equation. This transcendental equation does not have a direct algebraic solution, so Kepler (1609) made laborious successive approximations to solve for the true orbit of Mars. Newton's subsequent invention of calculus produced a more efficient method, based on successive derivatives, that rapidly converge on a true orbit solution. Obtaining true orbits is beyond the scope of this paper. Please refer to Rowe et al. (2024) and Romero (2024) for two computerized iterative solution techniques. The focus of the paper is entirely on Desmos' analysis of apparent orbits. Since the USNO's plots of a known binary is the starting point for this graphical analysis, a detailed description of these plots is in order.

2. Binary Star Plots from the 6th Orbit Catalog

The apparent orbit plot from the Sixth Orbit Catalog for the binary WDS 16212+2259, used as the example for Desmos analysis in this paper, is shown in Figure 3. The number of the binary in the *WDS Catalog* (WDS 16212+2259) is provided in the upper left corner. When this binary, previously thought to be a single star, was first reported in a publication to be a double star, it was assigned its WDS number based on its Right Ascension (RA) of 16 hours and 21.2 minutes, and Declination (Dec) of 22 degrees and 59 minutes. The discoverer code, HU 481, was also assigned (HU 481 for William Hussey's 481st discovery). In the upper right corner is a reference to the paper where the latest orbit was published, Hrt2010a (a paper by William Hartkopf in 2010). The "a" indicates that this was the first published orbit for this binary. Later orbital solutions would have been assigned b, c, d, etc., but since this is the only one listed, it is also the latest published orbital solution.

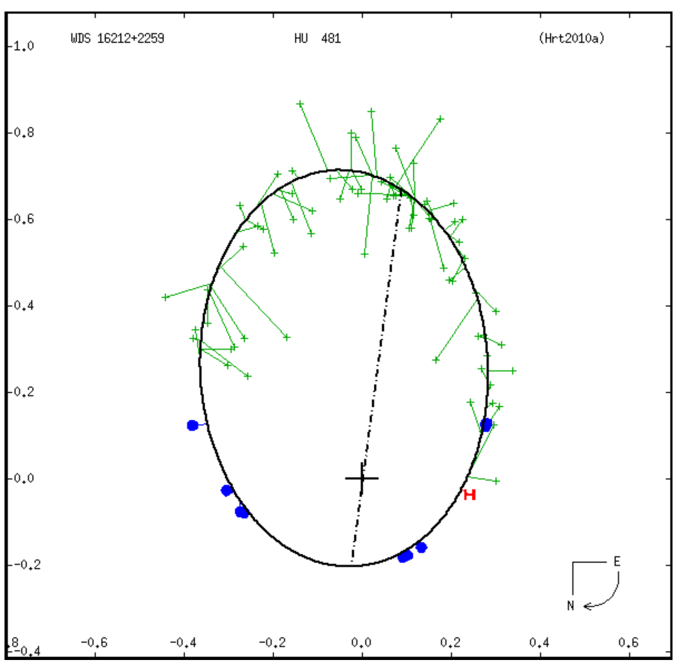

Figure 3: The apparent orbital plot of WDS 18044+0337 downloaded from the 6th Orbit Catalog.

The tick marks around the border of the chart are in arcseconds or fractions of an arcsecond. The horizontal and vertical zero points on the margins are aligned with the large **+** mark which is the location of the primary star. The measurements of the secondary (fainter star) are reported *relative* to the primary star, ignoring any actual motion in the sky of the primary star itself. This is done because it is much easier and more precise to measure relative positions between close stars rather than their absolute positions in RA and Dec.

The notation in the lower right corner of the plot provides the orientation of the plot with respect to celestial north; north being down and east to the right. The curved arrow shows the secondary star's direction of rotation relative to the "fixed" primary star, clockwise in this case.

The parameters that defined the true, face-on ellipse of the orbit were a best-fit calculation made by Hartkopf in 2010 based on the 84 published observations of this binary over 113 years from its discovery in 1902 to its (then) last reported observation in 2009. These parameter values, the *orbital elements,* are reported in the 6th Orbit Catalog. The black ellipse in the orbital plot is *not* the true face-on ellipse, but rather the reoriented (tilted) ellipse as it appears when viewed from Earth.

If Hartkopf's estimate of the orbital parameters was accurate, the observations of the secondary star over time would track the apparent ellipse around the primary star. While Hartkopf did the best he could in his 2010 paper to account for the then-published observations, subsequent observations (as we will see) deviated from his published ellipse. If several subsequent observations deviate from the ellipse in a coordinated manner, revising the orbital parameters to fit a new, better-fitting ellipse may be in order.

The black dashed line that passes through the primary star **+** is the *line of nodes* that shows where the plane of the true orbit intersects the plane of the apparent orbit. In other words, it is the hinge line that the true orbit is tilted about to obtain the apparent orbit.

Observations are color-coded. Each visual observation with filar a micrometer is shown as a **green +,** the **blue dots** are for speckle interferometry observations, while the **red H** is for a 1991 observation made by the European Space Agency's Hipparcos astrometric telescope.

Each observation is plotted as the angular distance of the secondary star from the primary star (their separation, r, in arcseconds) and the position angle of the secondary star (θ) with respect to the primary star and celestial north. Note the lines connecting the observational points to the black apparent orbit ellipse. These lines touch the ellipse at the point in time where the orbital equation predicts that the ~~the~~ secondary star should have been.

As can be readily concluded from the plot, the **green +** visual observations were not very accurate in comparison to the **blue dot** speckle observations and **red H** astrometry space telescope observations. This disparity was primarily due to three factors.

First, visual observations (**green +**) were limited by the capabilities of the human eye as a detector. The eye's exposure time is fixed and can't be made longer to gather photons for observing fainter stars, or shorter to avoid atmospheric-induced motion smearing. Also, the quantum efficiency of the human eye is only about 10%, while that of a good electronic camera these days is about 90%.

Second, the refractor telescopes used in early observations generally had smaller apertures than the reflecting telescopes used in later observations, thus having lower resolution. This made it difficult for close binary stars to appear as two distinct stars in smaller-aperture telescopes.

Third, and of greatest importance, advances in technology allowed the adverse effects of poor *seeing* to be greatly reduced or eliminated. Seeing … "is the degradation of the image of an astronomical object due to turbulence in the atmosphere of Earth that may become visible as blurring, twinkling, or variable distortion" (Wikipedia, 2023b). Seeing problems can be eliminated by placing telescopes above the atmosphere, as has been done with the Hipparcos (**red H**) and (more recently) the European Space Agency's Gaia astrometric telescopes. Placing telescopes in space is expensive, however. Anton Labeyrie (1970) showed that a series of very short exposures (just tens of milliseconds) could eliminate atmospheric smearing in ground telescope images. While each image was filled with speckles due to the different paths that light could take through the many small air cells of different temperatures and refraction, hundreds or thousands of short-exposure images could be processed mathematically via a fast Fourier transform to produce an image much like one would have obtained if there were no atmosphere. This process is termed speckle interferometry (**blue dots**).

Given these three factors, it is not surprising that large reflecting telescopes (equipped with modern electronic cameras making speckle interferometric measurements) can provide much more accurate astrometric measurements than visual micrometer measurements.

3. Desmos Background 6th Orbit Plot

To initiate the Desmos analysis of binary stars, a new graph is started or, better yet, an existing graph can be downloaded and used as a template. Starting with a binary star analysis template has the advantage that all the needed equations and other expressions are already in place. The graph for this example, WDS 18044+0337 is available at https://www.desmos.com/calculator/aeem8b6oj4. It might be noted that graphs created in Desmos can be tagged with a URL and shared with others (a permalink).

Once the graph is opened and given a new name, the next step is to past in a 6th Orbit plot as a background. In Desmos this can be done by clicking the large **+** sign in the upper left and selecting Image.

On their plots, Astronomers have assigned north (0°) as straight down, and all angles are measured from north clockwise (NESW). In the Cartesian plane, angles are usually measured from the positive x-axis. As Desmos operates in the traditional Cartesian plane, the angle of the background plot is set to 90°, rotating north so it lies along the positive x-axis. Be sure to click on the Graph Settings wrench icon (upper right) and click Degrees, as the default is in Radians).

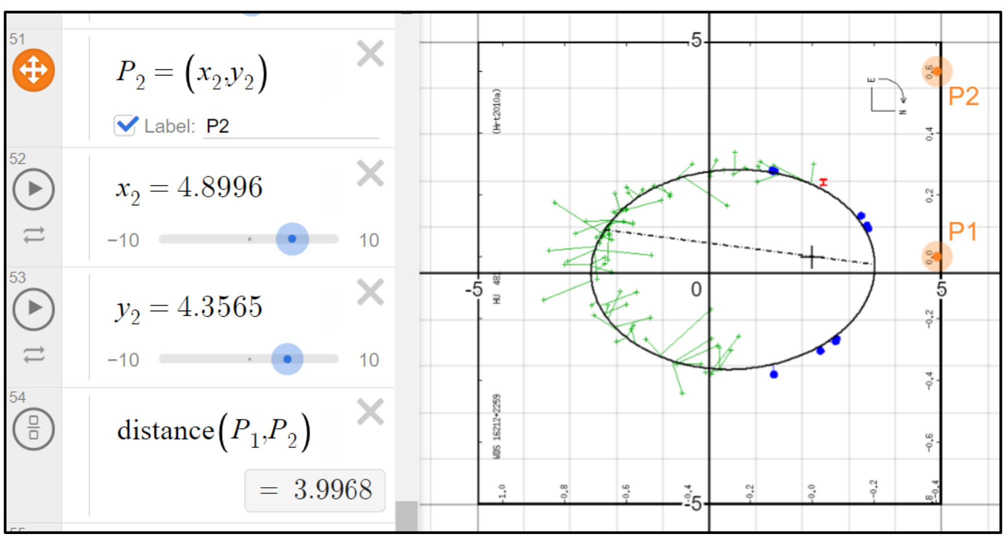

Figure 4: Using the movable pointers, P1 and P2, to measure the distance required for scaling.

The width and height of the background image plot is set by default as 10.0 Desmos units centered on the Desmos (0,0) origin as shown in Figure 4. The width and height both need to be equally reduced such that the Desmos scale (divisions) match those of the 6th Orbit plot. In the example, moveable points PI and P2 were moved to the 0.0 and 0.6 tick marks on the 6th Orbit plot, and the distance between them was found to be 3.9968 Desmos units. The precise placements of P1 and P2 were made with the graph highly magnified. This provided the information needed as two equal ratios: 10 Desmos units (the initial size of the background plot) is to the P1-P2 measured distance, as the final height (and width) value is to the 6th Orbit scale (tick marks). Thus, the final Desmos plot height (and width) = (10 Desmos units * 0.6 Orbit plot units)/3.9968 = 1.5012

Alternatively, clicking on the image will bring up the nine markers used to adjust the background. Any one of the four markers can be moved in or out to decrease or increase the size of the background image. The position of the primary star (the **+** on the orbital plot) can then be moved by dragging the center marker to position the **+** (primary star) on the plot to be precisely over the Desmos (0,0) coordinates. Again, magnification can aid precision. These two adjustments have to be done recursively as the movements interact with each other.

4. Washington Double Star Catalog Observations

The 6th Orbit plots often do not contain the most recent published observations. However, the *Washington Double Star Catalog,* maintained by the U.S. Naval Observatory, is a compilation of all published observations, including the first double star observations (Herschel 1785) as well as recent unplotted observations. Data on specific binaries can be requested from the Naval Observatory and supplied as a text file. In the analysis of binary stars, it is important to include recently published observations not yet appearing on 6th Orbit plots, as they are often the most helpful of all past observations in interpreting a just completed new observation.

The Washington Double Star Catalog (WDS) text files provided by the Naval Observatory consist of five sections: Measures, Orbital Elements, Orbital Ephemerides, Notes, and References. The Measures section can be copied into an Excel spreadsheet and (Data) Text-to-Columns applied. Unwanted information (both rows and columns) can then be deleted. For this example, just the WDS 16212+2259 Hipparcos and speckle interferometry observations were retained as shown in Table 1. A recent unpublished observation made on the 1.5-meter telescope at Mt. Wilson Observatory was added as the last row in the table. Also, an additional column (far left) was added with labels for the observer and year of observation. Bibliography codes in the References section can be pasted into a website (NASA ADS 2023) to download most of the references.

The most recent (June 2023) observation of WDS 16212+2259 is reported in this paper for the first time as the last row of Table 1. This observation, of course, was not included in the WDS Catalog text file received from the Naval Observatory. This binary was observed on the historic 1.5-meter telescope (once the world's largest) at Mt. Wilson Observatory. The instrumentation for the speckle interferometry observations and the initial data reduction technique using the Speckle Toolbox (Rowe & Genet 2915, Harshaw et al. 2017) have been documented by Faughn et al. (2023).

Table 1 column headers were added for clarification: Label identifies the observer and year, Date is the year and fraction of a year when the observation was made, Position Angle (θ°) is the angle of the secondary star (in degrees) relative to the primary star (**+**), Separation (Sep) is the distance between the two stars (r") in arcseconds, while Aperture (Ap in meters) is helpful in identifying the observational telescope. For instance, 2.5 meters is 100 inches, the famous 100-inch telescope Edwin Hubble used to discover the size and expansion of the universe, the Southern Astrophysical Research (SOAR) telescope in

Chile is 4.1 meters, while the telescope used at Mt. Wilson Observatory is 1.5 meters. The # is the number of nights averaged together to form the reported r and θ. The Reference (Ref) is where the observation was published, while Technique (T) is the type of measurement (ones starting with an “S” are speckle interferometry, while starting with H indicates a space telescope).

Table 1: Abbreviated extract from the WDS Catalog.

Label	Date	PA	Sep	Ap	#	Ref	T
Bla1986	1986.455	113.7	0.3030	3.6	1	Bla1987	S
Bla1986	1986.455	114.7	0.3060	3.6	1	Bla1987	S
HIP1991	1991.250	81.0	0.2450	0.3	1	HIP1997a	Hh
Hrt1995	1995.438	39.7	0.2070	2.5	1	Hrt1997	Sc
Hrt1996	1996.323	29.5	0.2040	2.5	1	Hrt2000a	Sc
Hrt1996	1996.539	26.6	0.2030	2.5	1	Hrt2000a	Sc
Hrt2007	2007.313	286.5	0.2770	2.5	1	Hrt2009	Su
Gii2008	2008.390	285.7	0.2840	0.7	1	Gii2012	S
Tok2009	2009.262	275.1	0.3065	4.1	1	Tok2010	S
Gii2014	2014.524	253.1	0.3850	0.8	1	Gii2022	S
Tok2015	2015.337	252.1	0.4010	4.1	2	Tok2016a	St
Tok2018	2018.399	243.6	0.4446	4.1	2	Tok2019c	St
Tok2021	2021.319	237.2	0.4812	4.1	2	Tok2022f	St
MWO2023	2023.489	233.2	0.5170	1.5	1	This Paper	S

WDS observations are given as the apparent separation between the primary and secondary stars in arcseconds (r), and the position of the secondary star with respect to celestial north in degrees (θ). Note that θ is reserved in Desmos for angles. The separation and position angle for each observation, r, and θ, are entered, as well as a label for the point, such as the three-letter observer designator and year of observation (e.g., Tok2018). Desmos does not directly plot points in polar coordinates, so the (x,y) position coordinates are calculated for the ith added (x_i, y_i) point as ($r_i \cos \theta_i$, $r_i \sin \theta_i$).

When labels for points are entered in Desmos, it automatically places them at (usually) a good location, although their location can be changed for added clarity. Points and their corresponding labels can also be color-coded and presented as a filled circle, open circle, or x marks the spot to help avoid confusion. Besides entering the coordinates and labels of new observations, it can also be useful to label some already included observations on the background, such as accurate speckle interferometry or space observations, as these labels can help clarify the analysis.

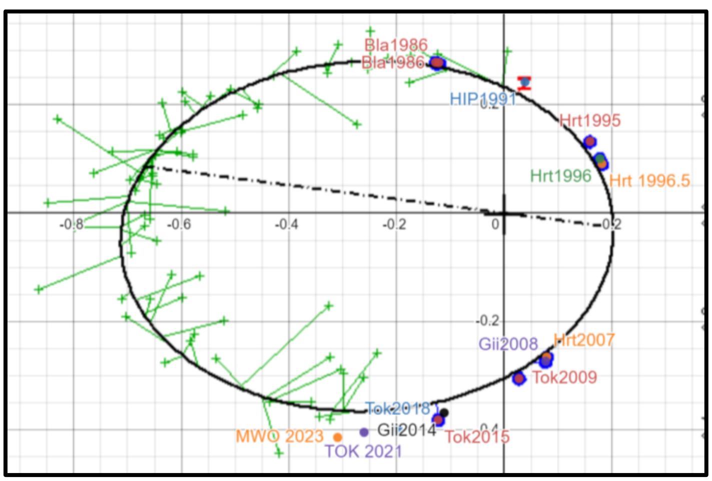

Figure 5: Three new and 12 past observations plotted and labeled.

Three new speckle observations (MWO 2023, Tok2021, and Tok2019), 12 past speckle observations, and one space observation have been added and labeled as shown in Figure 5. MWO2023 was a recent observation made by students and supporters on the 1.5-meter telescope at Mt. Wilson Observatory in

June of 2023. If the 6th Orbit plot background image was precisely scaled and translated then, when the coordinates are entered for previously plotted observations, the observations will show up as small dots in the center of the large blue dots on the plot.

5. General Ellipse Equations

The general equation for the x and y values of an ellipse centered on the origin with its major axis coincident with the x-axis is:

$x^2/a^2 + y^2/b^2 = 1$

where:
a is the semi-major axis
b is the semi-minor axis

For an ellipse not centered on the origin and not coincident with the x-axis, as is the binary star case, one can use the general parametric equations for an ellipse in the Cartesian plane:

x = (h + a cos t)(cos q) + (k + b sin t)(-sin q)
y = (h + a cos t)(sin q) + (k + b sin t)(cos q)

where:
a is the semi-major axis
b is the semi-minor axis
q is the angle from the x-axis to the ellipse major axis
h is the offset of the center of the ellipse from the primary star (the "**+**") along the ellipse's primary axis *after* the ellipse has been rotated
k is the y offset of the center of the ellipse from the primary star (the "**+**") perpendicular to the ellipse's primary axis *after* the ellipse has been rotated
(t is just the parametric variable that traces out the ellipse from 0° to 360°)

The (x,y) coordinates of the ellipse center are:

(h cos q – k sin q, h sin q + k cos q)

This is shown graphically in Figure 6 where the line segment from the Primary Star to the First Offset is from (0,0) to (h cos q, h sin q) is of length h, and the segment from the First Offset to the Ellipse Center is [(h cos q, h sin q), (h cos q – k sin q, h sin q + k cos q)] and is of length k. The line drawn through the center of the ellipse at angle q is the major axis of the ellipse. Note that it does *not* pass through the Primary Star at (0,0).

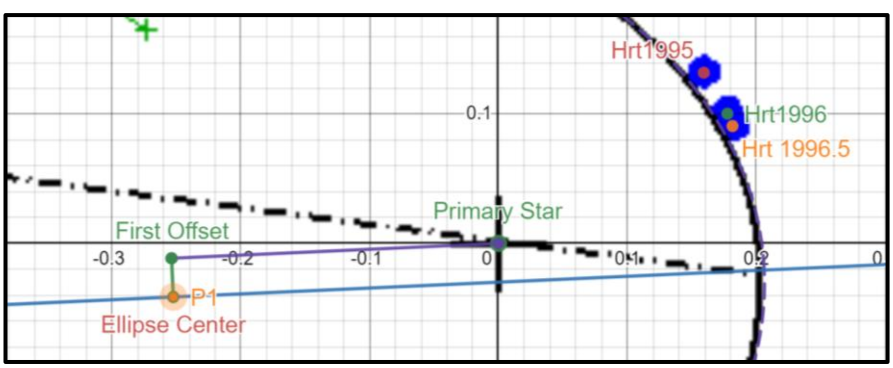

Figure 6: Ellipse center with respect to the h and k offsets at angle q.

6^{th} Orbit Apparent Ellipse Equation

To obtain the five ellipse parameters for the 6^{th} Orbit apparent ellipse, the five parameter sliders can be adjusted iteratively until an ellipse (small red dots) in Figure 7 overlays the black line of the 6^{th} Orbit published ellipse.

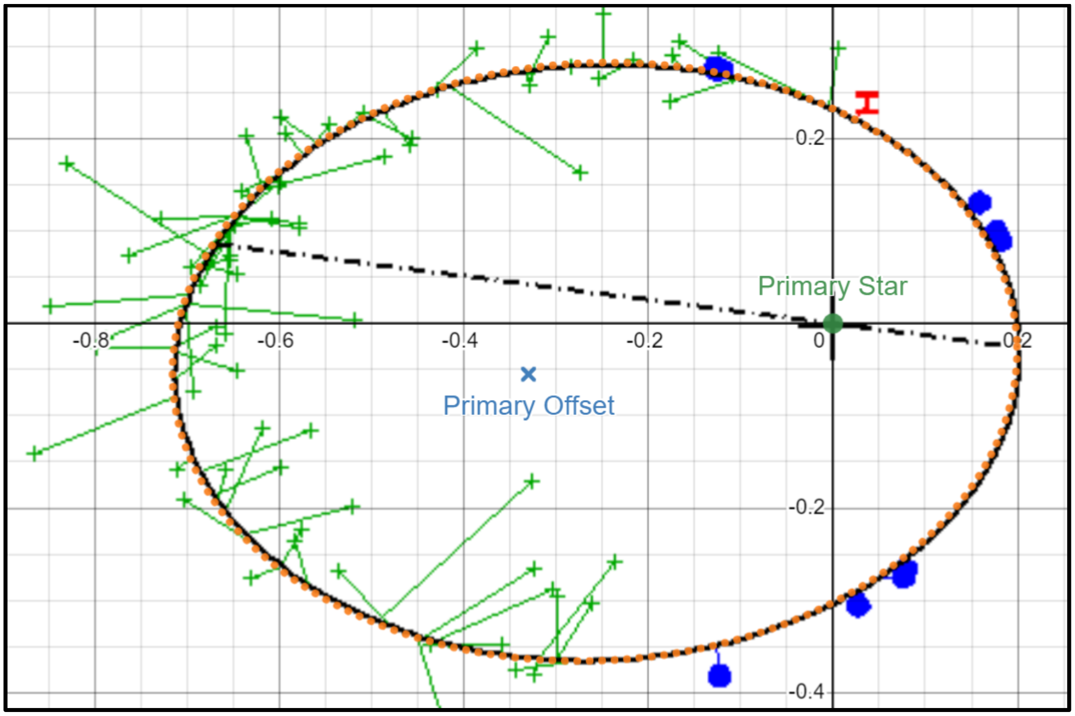

Figure 7: Ellipse (red dots) overlaid on the solid black line 6^{th} Orbit ellipse.

An easier and more accurate way to obtain the apparent ellipse parameters is to analytically derive them from the true Orbital Elements published in the 6^{th} Orbit Catalog (included in the WDS data supplied by the USNO on individual binaries). The new Orbits routine (Rowe & Genet 2024) in the Speckle Toolbox derives the apparent ellipse parameters from the true orbit parameters. The semi-major and semi-minor axes, as well as the rotation angle of the ellipse, can be entered directly from Orbits into Desmos, but the x,y offsets in Orbits have to be translated into the h,k Desmos offsets at the angle (q in Desmos) by:

$h = x \cos q + y \sin q$
$k = y \cos q - x \sin q$

6. New Apparent Ellipse

The apparent orbit shown in Figure 7 (black ellipse overlaid with the red dot ellipse) was calculated by Hartkopf (2010) without the benefit of the five speckle observations made since his analysis. Given this new information, a new apparent ellipse was created in Desmos that fits the speckle interferometry points better than Hartkopf's 2010 orbit. Starting with the 6^{th} Orbit apparent ellipse, the five parameters were adjusted to fit an ellipse to the speckle and space observations. The visual observations (green +) were not considered while making this fit as their accuracy was low for this small-separation binary. The result is shown in Figure 8. As can be seen (and expected) the new ellipse provides a significantly better fit to the speckle observations.

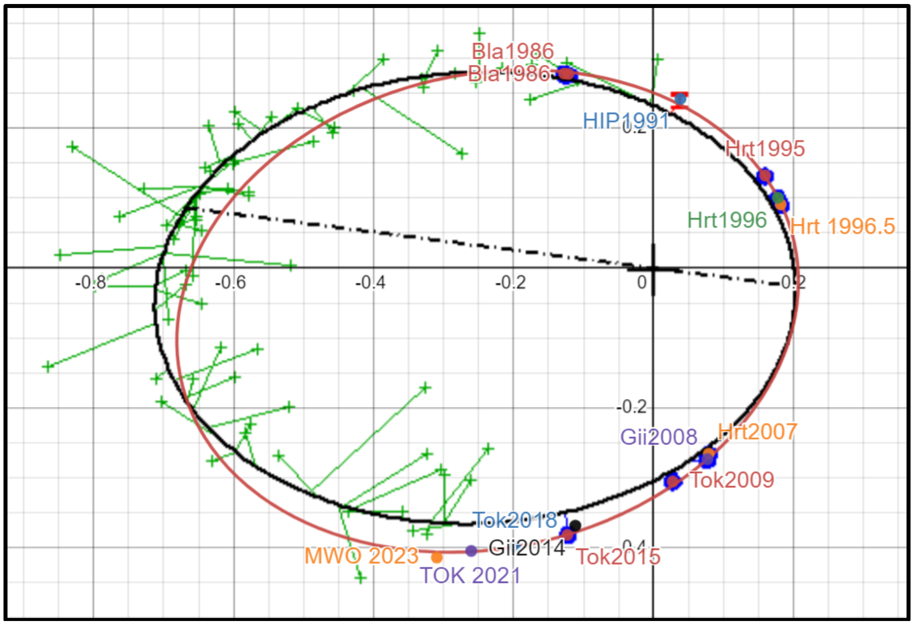

Figure 8: Apparent ellipse fitted to speckle interferometry and space observations.

Table 2 lists the values for the five ellipse parameters for both the original (black) apparent ellipse and the new (red) apparent ellipse.

Table 2: Ellipse parameter values for WDS 16212+2259 new orbit and 6th Orbit Catalog original ellipse.

Parameter	Name and Units	6th Orbit	New
a	semimajor axis "	0.458	0.456
b	semi-minor axis "	0.324	0.335
h	primary star x offset	-0.260	-0.250
k	primary star y offset	-0.028	-0.008
q	angle of major axis °	3.000	14.300

If the recent observations added in the Demos analysis do not suggest a systematic departure from the published orbit, then the Desmos apparent orbit analysis is completed. If, as is the case here, there is a significant, systematic departure from the published orbit, then a new orbit could be calculated, a procedure beyond the scope of this paper.

7. Conclusion

Desmos, combined with plots from the 6th Orbit Catalog and observational values from the Washington Double Star (WDS) Catalog can be used to analyze the apparent orbits of known binaries. Desmos combines mathematical precision with intuitive visual displays.

Acknowledgments

The United States Naval Observatory supplied past observations of WDS 16212+2259 from their *Washington Double Star Catalog*. The background plot was from the *Sixth Catalog of Orbits of Visual Binary Stars*. The Institute for Student Astronomical Research (InStAR) purchased time on the 1.5-meter telescope at Mt. Wilson Observatory where the new speckle interferometry of WDS 16212+2259 was made. Gravic Inc. provided travel funds for students to attend the run. Rachel Freed, Reed and Chris

Estrada, Nick Hardy, Leon Bewersdorff, Joseph Burch, Paul McCudden, Tom Smith and others helped organize the run and managed the observations. Thomas Meneghini, the Director of the Mt. Wilson Observatory, and his staff provided support during the run. We thank several external reviewers for their helpful comments.

References

Desmos. 2023. https://www.desmos.com/calculator

Faughn, Elias, John Major, & Paul McCudden. 2023. Speckle Astrometry of WDS 18181-0120. *Journal of Double Star Observations,* 19, 421.

Harshaw, Richard, David Rowe, & Russell Genet. 2017. The Speckle Toolbox: A Powerful Data Reduction Tool for CCD Astrometry. *Journal of Double Star Observations,* 13, 52.

Hensley, Hagan. 2018. The Double Star Orbit Initial Value Problem. *Journal of Double Star Observations,* 14, 363.

Herschel, W. 1785. Catalog of Double Stars. *Philosophical Transactions of the Royal Society of London,* 75, 24.

Herschel, W. 1803. Account of the changes that have happened, during the last twenty-five years, in the relative situation of double stars. *Philosophical Transactions of the Royal Society* **93**, 339.

Kepler, Johannes. 1609. *Astronomia Nova*. 2nd translated edition by William Donahue, Green Lion Press.

Labeyrie, A. 1970. Attainment of diffraction limited resolution in large telescopes by Fourier analyzing speckle patterns in star images. *A&A,* 6, 85.

Matson, Rachel A., S. J. Williams, W. I. Hartkopf, & B. D. Mason. 2023. *Sixth Catalog of Orbits of Visual Binary Stars,* U.S. Naval Observatory, Washington, DC. http://www.astro.gsu.edu/wds/orb6.html

NASA ADS. 2023. https://ui.adsabs.harvard.edu/

Rowe, David, and Russell Genet. 2015. User's Guide to PS3 Speckle Interferometry Reduction Process. *Journal of Double Star Observations,* 11 (1s), 266.

Rowe, David, and Russell Genet. 2024. Speckle Toolbox Orbits Analysis: WDS 21001+0731 Example. *Journal of Double Star Observations,* in press.

Washington Double Star Catalog. 2023. United States Naval Observatory, http://www.astro.gsu.edu/wds/

Wikipedia 2023a, Definition of astrometry, https://en.wikipedia.org/wiki/Astrometry

Wikipedia 2023b, Definition of seeing, https://en.wikipedia.org/wiki/Astronomical_seeing