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Desmos Analysis of the Binary Star CHR 259 (WDS 15496-0326)

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Abstract Desmos, an online graphing calculator was used to analyze observations of the Binary Star WDS 15496-0326 (CHR 259), including a speckle interferometry observation first reported in this paper obtained with the 1.5-meter telescope at Mt. Wilson Observatory on 2023.4889 that measured a position angle of 309.01° and a separation of 0.470". Since this observation and one other recent observation closely aligned with Tokovinin's 2018 published orbit, there is no need to recalculate the orbit at this time.

Introduction

New speckle interferometry observations of the binary star WDS 15496-0326 (CHR 259) were made by students, instructors, and astronomers at Mt. Wilson Observatory in June of 2023. The Desmos online graphing calculator was used to analyze the published observations of CHR 259 and the new observation from Mt. Wilson. These observations were compared with the apparent orbital plot from the USNO Sixth Catalog of Orbits of Visual Binary Stars (hereafter the 6th Orbit Catalog) to determine if it would be worthwhile to calculate a new orbit (Matson et al. 2023).

The Desmos analysis in this paper is located at https://www.desmos.com/calculator/ofmie1xfh3. Desmos accounts are free. Anyone can start an account and download this analysis to their account.

Discovery of HU 256 as a Double Star

Astrometry "is a branch of astronomy that involves precise measurements of the positions and movements of stars and other celestial bodies" (Wikipedia 2024). Obtaining highly accurate positions for stars with telescopes located on Earth is hampered by atmospheric distortions (seeing). To overcome this difficulty, the European Space Agency launched the Hipparcos astrometric telescope in 1989 (Peryman 2010).

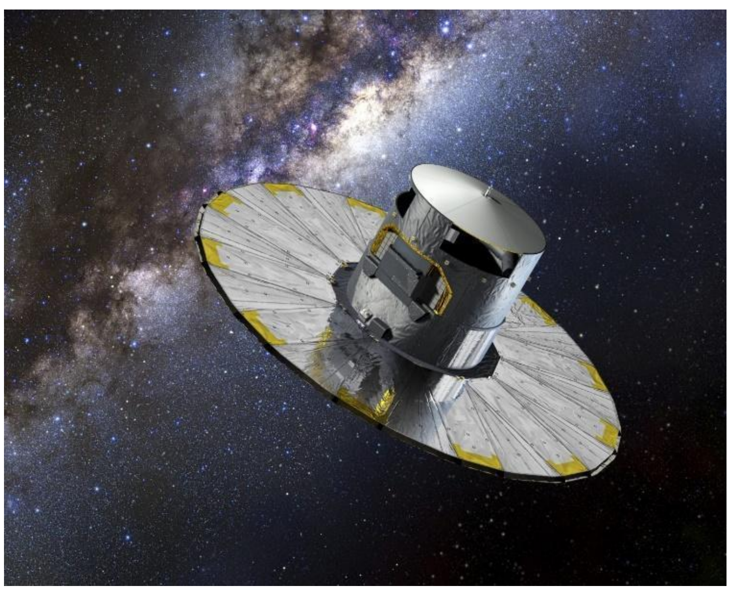

Figure 1: The European Space Agency's Hipparcos astrometric space telescope operated between 1989 and 1993, gathering precise data on the positions and radial velocities of over 100,000 stars.

Hipparcos is an acronym for High Precision Parallax Collecting Satellite, and also a reference to the Greek astronomer Hipparchus (190-120 BC) who discovered the precession of the equinoxes. Hipparcos operated from 1989-1993, scanning the sky multiple times with its two telescopes. It obtained the accurate positions and radial velocities of ~ 118,200 nearby stars in our galaxy.

Each star was observed multiple times to increase precision by averaging out small measurement errors. There were, however, several stars with changes in position over the three years that were larger than just measurement errors. It seemed likely that at least some of these “problem stars” might not be single stars, but were binary stars with the apparent separations between the two stars so small that Hipparcos could not resolve them as two separate stars. Perhaps their photocenters (apparent locations of the unresolved stars) were in motion. WDS 15496-0326 (CHR 259) was one of these problem stars.

Hipparcos, with an aperture of only 0.3 meters, didn’t have sufficient resolution to determine if these problem stars were two stars, let alone make the many observations over a number of years it would take to determine if these were gravitationally bound binary stars rotating around a common center of gravity. However, much larger Earth-based telescopes might be able to resolve many of these stars if they overcome the effects of atmospheric turbulence via speckle interferometry. Speckle interferometry overcomes atmospheric distortion by taking many hundreds of exposures so short (typically tens of milliseconds) such that atmospheric jitter is essentially frozen out (Genet 2015). A complex, computerized mathematical technique, bispectrum analysis, is then used to synthesize an image much like one would see from space via a deconvolution process that uses similar observations of a nearby single star to remove optical distortions.

The three years of the Hipparcos astrometric space telescope observations were completed in 1993, and were used to calculate stellar positions and radial velocities with a central (as of) date of 1991.25. Although the analysis was not completed and published as a catalog until 1997 (Fabricius & Makarov 2000) Hipparcos 1997), lists of some of the problem stars that included CHR 259 began circulating by 1993. Astronomers with access to large-aperture telescopes instrumented for speckle interferometry lost no time in observing these problem stars to determine if they were double stars. In early 1995, Brian Mason et al. (1999) were the first to obtain a speckle interferometry observation of CHR 259, using the historic 2.5-meter Hooker telescope at Mt. Wilson Observatory. They discovered that it was indeed a double star.

Establishing HU 256 as a Gravitationally Bound Binary

While a single speckle interferometry observation can discover that a star, previously thought to be a single star is; in fact, a double star, determining whether or not it is a gravitationally bound binary star takes many observations over years, decades, or even centuries, depending on the length of the binary’s period. Besides their discovery observation, the Mason et al. (1999) team obtained three more speckle observations of HU 256 between 1995 and 1997. Mason et al. (2001) then observed CHR 259 with the 2.1-meter telescope at McDonald Observatory in 1999, while Lewis Roberts and Mason (2018) used adaptive optics on the 3.6-meter telescope on the summit of Haleakalā on Maui for a 2003 observation. Back on the 2.5-meter telescope at Mt. Wilson Observatory in 2006, William Hartkopf and Mason (2009), obtained another speckle interferometry measurement. Andre Tokovinin et al. (2010) made two measurements of CHR 259 on the SOAR (Southern Astrophysical Research) 4.1-meter telescope in Chile in 2009. Jack Drummond (2014) obtained three observations in 2011 using the 3.5-meter telescope equipped with adaptive optics at the Starfire Optical Range at Kirtland Air Force Base near Albuquerque, New Mexico. Andre Tokovinin et al. (2015, 2018) contributed two more SOAR observations in 2014 and 2016. At this point, Tokovinin (2018) concluded that there were sufficient observations to calculate and publish an orbit, officially establishing HU 256 as a binary.

Tokovinin et al. (2019, and 2022) contributed two more speckle interferometry observations in 2018 and 2021 that were not included in the orbit published in the 6th Orbit Catalog, but were included as published observations in the Washington Double Star Catalog (2023). These two observations rounded out the currently published observations. These published observations are shown in Table 1. Label is the observer (first author) and year of observation, Date is the year and fractional year, PA is the position angle (°), Sep is the separation ("), Ap is the aperture of the telescope in meters, Ref is the published reference code, and Tq is the technique used for the observation (Hh is Hipparcos, Sc, Su, S, and St are all various types of speckle interferometry, while A is adaptive optics.

Table 1: Published observations of WDS 15496-0326 (CHR 259).

Label	Date	PA	Sep	Ap	N	Ref	Tq
Hip1991	1991.2500	106	0.2000	0.3	1	Fab2000b	Hh
Hrt1995	1995.1432	91.2	0.2770	2.5	1	Hrt1999d	Sc
Hrt1996	1996.4319	86.3	0.2620	2.5	1	Hrt1999d	Sc
Hrt1996	1996.6998	86.3	0.2540	2.5	1	Hrt1999d	Sc
Hrt1997	1997.4585	80.8	0.2540	2.5	1	Hrt1999d	Sc
Msn1999	1999.1667	71.5	0.2290	2.1	2	Msn2001b	Su
Rbr2003	2003.5069	65.1	0.1000	3.6	1	Rbr2018	A
Hrt2006	2006.5595	14.6	0.1590	2.5	1	Hrt2009	Su
Tok2009	2009.2643	353.5	0.2059	4.1	1	Tok2010	S
Tok2009	2009.2643	353.6	0.2051	4.1	1	Tok2010	S
Rbr2009	2009.2648	356.7	0.1934	5.1	1	Rbr2013a	Ac
Dru2011	2011.2180	342.9	0.2330	3.5	2	Dru2014	A
Dru2011	2011.3197	340.9	0.2360	3.5	1	Dru2014	A
Dru2011	2011.4730	341.4	0.2410	3.5	1	Dru2014	A
Tok2014	2014.1844	329.3	0.2897	4.1	2	Tok2015c	St
Tok2016	2016.1376	323.3	0.3285	4.1	2	Tok2018c	St
Tok2018	2018.1798	318.5	0.3685	4.1	2	Tok2019c	St
Tok2021	2021.3188	312.9	0.4261	4.1	2	Tok2022f	St

Recent Observation of CHR 259

As part of the Known Binaries Program (Hardy et al. 2023), speckle interferometry observations were obtained for 50 close binaries on the 1.5-meter telescope at Mt. Wilson Observatory in June 2023 by a group of students, instructors, and amateur and professional astronomers (Faughn et al. 2023). Included was an observation of WDS 15496-0326 (CHR 259). The Speckle Toolbox (Rowe and Genet 2015, Harshaw et al. 2017) was used to reduce the observations of CHR 259 and a nearby single reference star, resulting in a Position Angle of 309.01° and a Separation of 0.470" on 2023.4889. The bispectrum reconstructed image of the binary obtained at Mt. Wilson Observatory is shown in Figure 2.

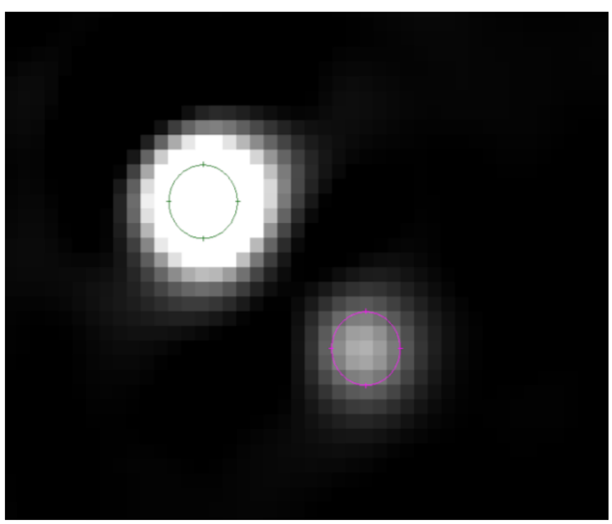

Figure 2: Image of CHR 259 reconstructed via bispectrum analysis of hundreds of short exposures of the

binary and a nearby single reference star used for deconvolution. Each pixel is only 0.03" in width.

Desmos Analysis

The starting point for the Desmos analysis of CHR 259 was the orbital plot of CHR 259 downloaded from the Sixth Catalog of Orbits of Visual Binary Stars (Matson et al. 2023, hereafter the 6th Orbit Catalog). This plot is shown in Figure 3.

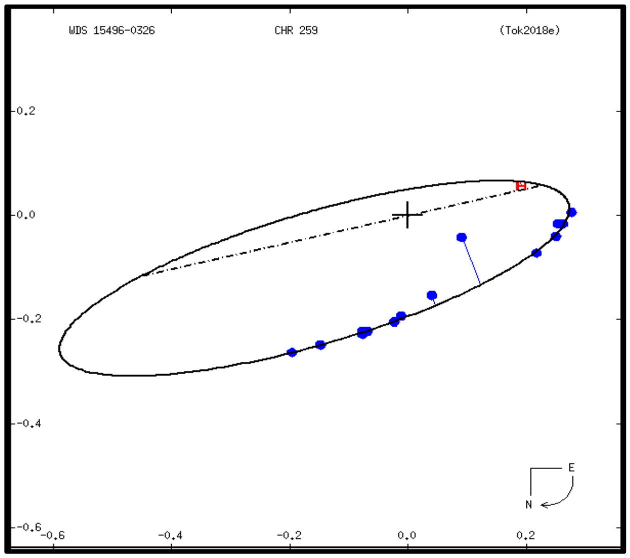

Figure 3: Orbital plot of CHR 259 from the 6th Orbits Catalog.

The 6th Orbit plot was inserted as a background image in Desmos. Following the procedure outlined by Genet et al. (2024), the image was rotated 90° counterclockwise to place celestial north along the positive x-axis. The background image scale (width and height), which in Desmos is a default of 10.0 units, was rescaled to 1.027 units to match the scale of the 6th Orbit plot, and the primary star location (the + on the 6th Orbit plot was translated to 0.122 units in x and -0.154 in y to be coincident with the Desmos (0,0) Cartesian origin. The final rotated, scaled, and translated 6th Orbit background plot is shown in Figure 4.

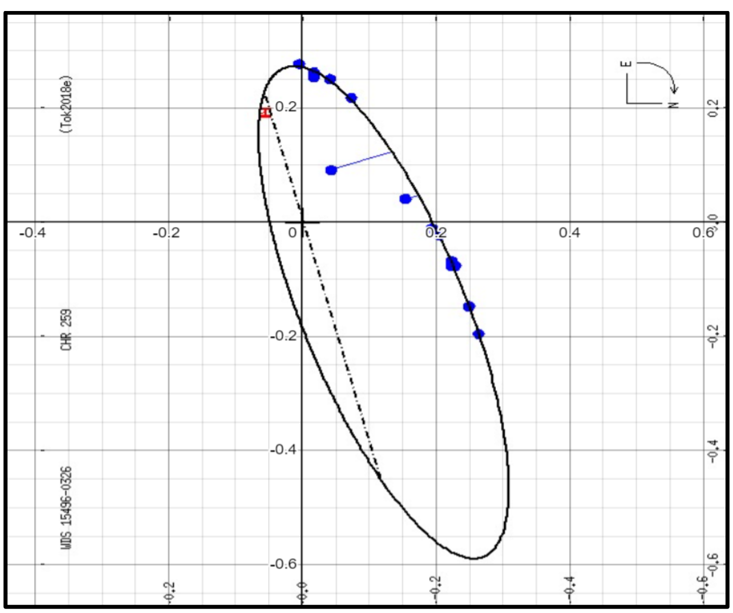

Figure 4: Appropriately rotated, scaled, and translated 6th Orbital plot of HU 259.

All the past observations shown in Table 1, as well as the recent observation at Mt. Wilson Observatory, were labeled, again following the procedures in Genet et al. (2024). These added labels are shown in Figure 5 (left), while a highly magnified view is shown on the right where it can be seen how each Desmos plotted point (red x) is in the center of the 6th Orbit plot's round blue points, demonstrating that the 6th Orbit background plot is properly registered with Desmos Cartesian coordinate system.

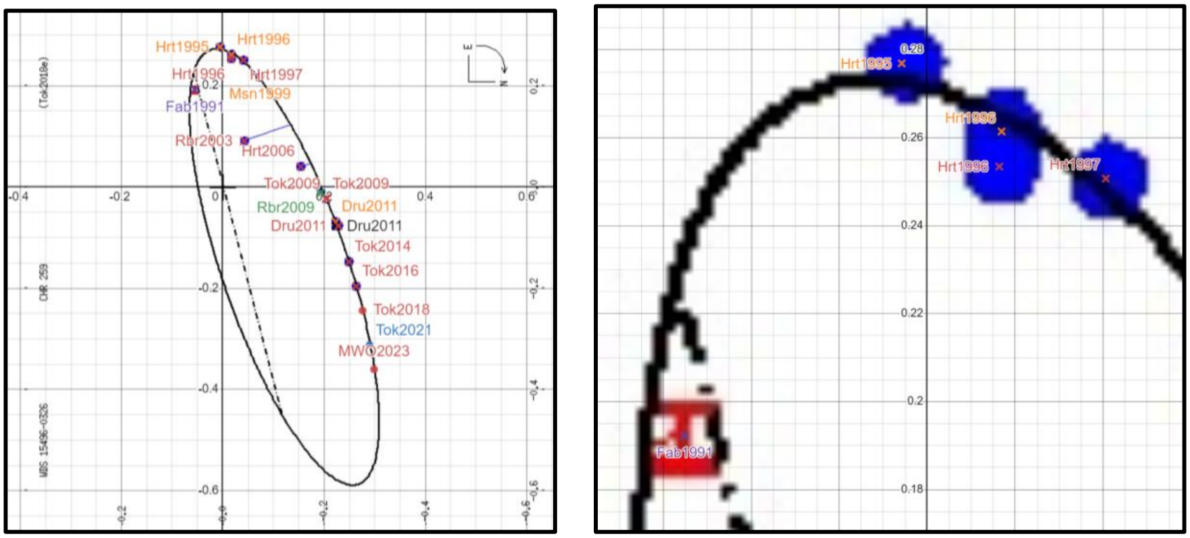

Figure 5: Orbital plot with added labels (left) and magnified image showing registration of Desmos points with the background.

For an ellipse not centered on the origin and not coincident with the x-axis, as is the binary star case, one can use the general parametric equations for an ellipse in the Cartesian plane:

$$x = (h + a \cos t)(\cos q) + (k + b \sin t)(-\sin q)$$
$$y = (h + a \cos t)(\sin q) + (k + b \sin t)(\cos q)$$

where:

a is the semi-major axis

b is the semi-minor axis

q is the angle from the x-axis to the ellipse major axis

h is the offset of the center of the ellipse from the primary star (the "+") along the ellipse's primary axis *after* the ellipse has been rotated

k is the y offset of the center of the ellipse from the primary star (the "+") perpendicular to the ellipse's primary axis *after* the ellipse has been rotated

(t is just the parametric variable that traces out the ellipse from 0 to 360°)

These two equations, which form the (x,y) points for an ellipse, were entered into Desmos. The first part of this equation can be seen upper left in Figure 6. The five parameters are displayed by Desmos on the left so that values can be entered directly if known, or sliders adjusted as desired. The sliders were iteratively adjusted to fit the 6th Orbit background ellipse. The Desmos ellipse (orange dots) overlays the black background 6th Orbit ellipse.

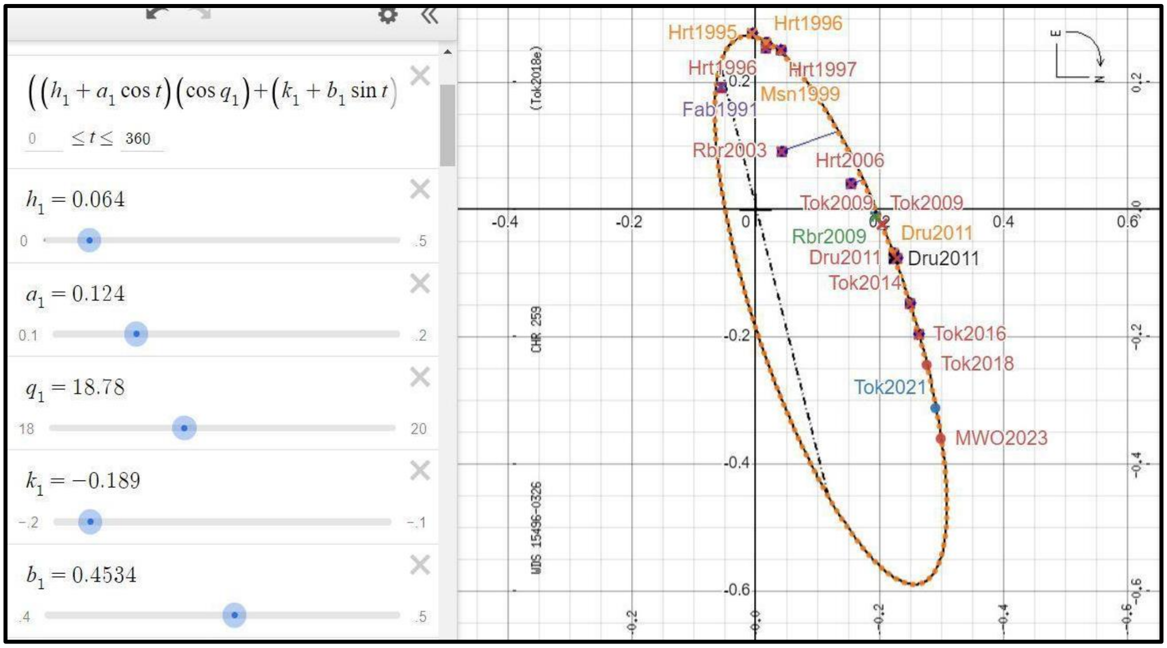

Figure 6: Orbital plot of WDS 15496-0326 from the 6th Orbit Catalog with matching Desmos plot and observation points including three unpublished observations.

Finally, while the recent observations seemed to follow the Tokovinin orbit published in 2018, a new orbit (green line in Figure 7 left) was created in an attempt to obtain an even better fit. An enlarged view (Figure 7 right) suggests that Tokovinin's 2018 orbit is a good fit to the speckle observations over the past decade, including three new ones made since the orbit was published. It is possible that future observations might follow the new green-line orbit. If they do, the orbital period would be somewhat shorter, and hence the combined mass of the two stars somewhat greater than currently estimated. Continued observations over the next decade should settle this question.

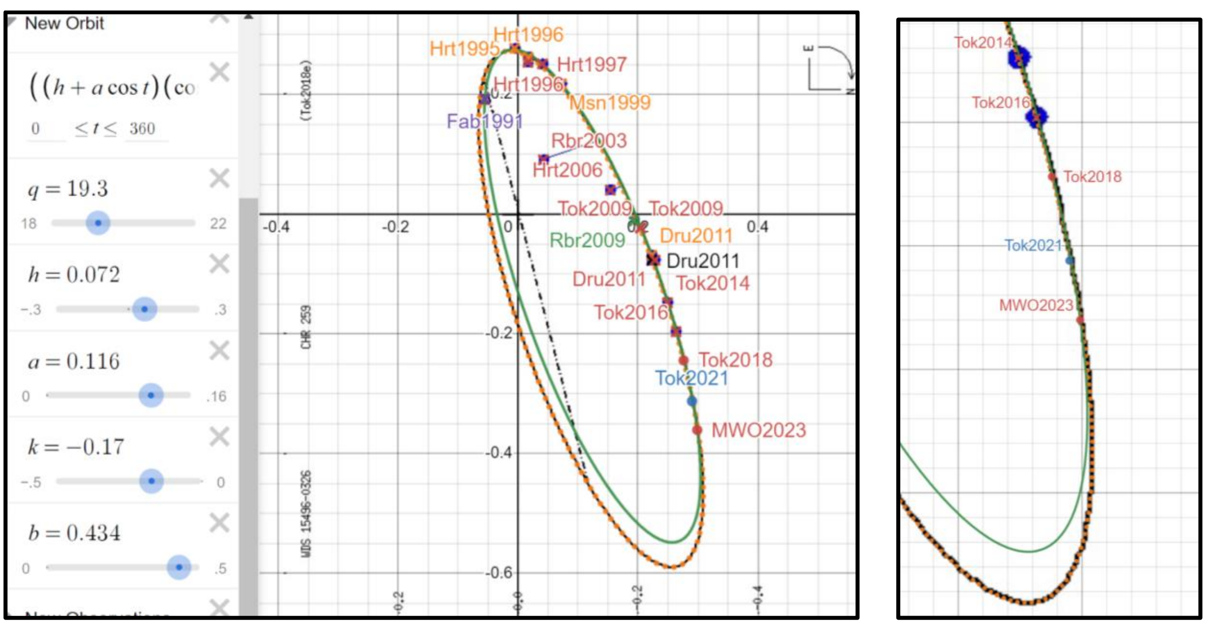

Figure 7: New orbital plot (green line, left), and magnified view (right).

Conclusions

WDS 15496-0326 (CHR 259), previously thought to be an unremarkable single star, was identified in 1994 as a “problem star” by the Hipparcos astrometric space telescope, and discovered, via speckle interferometry observations on the historic 2.5-meter telescope at Mt. Wilson Observatory, to be a double star. After 21 years of speckle interferometry observations on six different large-aperture mountaintops telescopes, Andre Tokovinin’s published orbit in 2018 officially confirmed CHR 259 as a gravitationally bound binary pair.

Since then, Tokovinin obtained two more speckle observations, and one of us (Genet), along with others, obtained a recent speckle observation of this binary on the historic 1.5-meter telescope also on Mt. Wilson Observatory, just yards away from the discovery 2.5-meter telescope. Reduction of this observation (this paper) yielded a position angle of 301.01° and separation of 0.470" on 2023.4889.

Desmos analysis of all 18 previously published observations and the one observation described in this paper conform to Tokovinin’s 2018 published orbit, hence no new orbit needs to be calculated at this time.

Acknowledgments

The United States Naval Observatory supplied past observations of WDS 16212+2259 from their *Washington Double Star Catalog*. The background plot was from the *Sixth Catalog of Orbits of Visual Binary Stars*. We utilized Dave Rowe’s Speckle Tool Box and the Desmos graphing calculator for reduction and analysis. The Institute for Student Astronomical Research (InStAR) purchased time on the 1.5-meter telescope at Mt. Wilson Observatory where the new speckle interferometry of WDS 16212+2259 was made. Gravic Inc. provided travel funds for students to attend the run. Rachel Freed, Reed and Chris Estrada, Nick Hardy, Leon Bewersdorff, Joseph Burch, Paul McCudden, Tom Smith, and others helped organize the run and manage the observations. Thomas Meneghini, the Director of the Mt. Wilson Observatory, and his staff provided support during the run. We thank Payson High School and Eastern Arizona College for their support of this analysis. Finally, we thank several external reviewers for their helpful comments.

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